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Waves induced by the longitudinal impact of a rod against a stepped rod in contact with a rigid barrier☆

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ABSTRACT

The longitudinal elastic central impact of a rod system, consisting of a uniform rod having a pre-impact velocity against a stepped rod which is in a state of rest and interacts by means of unilateral constraints with a rigid barrier, is modelled.

The problem of the longitudinal impact of a rod of distributed mass and a description of the motion of the cross-sections by wave equations was formulated by Navier, Boussinesq, Saint Venant and Sears.

In the second half of the twentieth century, the use of impact technologies in machine construction, the mining industry, building and instrument making led to a considerable number of theoretical and experimental investigations in the field of longitudinal impact. The collision of uniform rods of different length and cross-section areas was considered without taking account of possible repeated collisions in sections with unilateral constraints.[1,2] Repeated collisions in sections with unilateral constraints were taken into account in the process of longitudinal elastic and elastoplastic impact.[3–5] A relation was established between the duration of the impact and the ratio of the masses of the impacting body and the rigidly clamped rod. Mass ratios were found for which a secondary collision is observed. However, no account was taken of the effect of the configuration of the colliding elements on the impact process. The duration of the breaking of contact before repeated collision could not be successfully measured. The impact of a thin rod on a rigid Barrier has been considered in detail as well as the impact of two bodies, including the change in the velocity parameters of the sections and the stresses and strains of the colliding bodies.[6] However, the possibility of a repeated collision which arises as a result of the transformation of the shock waves and the effect of a repeated impact on the change in the magnitudes of the strains and stresses have not been fully revealed.

It can be seen that, in none of the papers mentioned, is there a qualitative estimate of the effect of repeated impacts on the dynamic characteristics of the impact process (the stresses and strains of greatest modulus developed in the elements of the impact system) or a systemization of the results obtained.

A model of the longitudinal impact of rods is presented below in which there are discontinuities in the constraints and in which repeated collisions occur.[7,8] An estimate of the effect of repeated impacts in sections with unilateral constraints on the change in the maximum longitudinal deformation of the uniform segments of the rod system is given.

1. Formulation of the problem

We will consider a mathematical model of the longitudinal impact of a uniform rod of mass m_1 and length l_1, moving at a velocity V_0, on a fixed non-uniform stepped rod. The length of the initial segment of the fixed rod is l_2, the length of the final segment is l_3 and the mass of the two segments is m_2. This fixed rod is in contact with a rigid barrier (Fig. 1). The overall length of the two rods is equal to l. All the segments are made of the same material. The wave model of a longitudinal impact[1,2,7,8] is used.

The motion of the cross-sections of the colliding rods is described by the wave equation

$$u_{nxx}(x,t) - \frac{1}{a^2}u_{ntt}(x,t) = 0, \quad n = \begin{cases} 1 & \text{when } 0 \le x \le l_1 \\ 2 & \text{when } l_1 \le x \le l_1 + l_2 \\ 3 & \text{when } l_1 + l_2 \le x \le l \end{cases} \tag{1.1}$$

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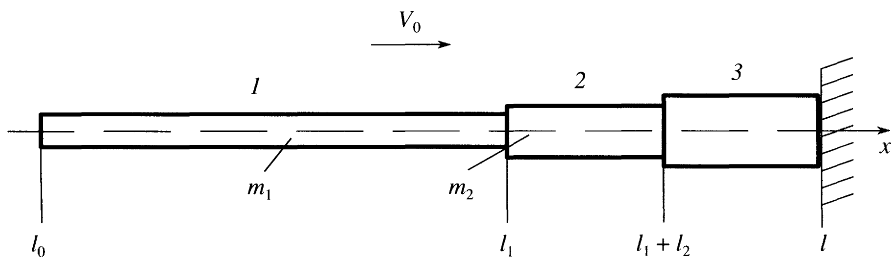

Fig. 1.

where $u_1(x, t)$, $u_2(x, t)$ and $u_3(x, t)$ are the longitudinal displacement of a cross-section of the uniform rod *1*, of the initial segment of the stepped rod *2* and of the final segment of the stepped rod *3* respectively, x is the coordinate of the cross-section, t is the time and a is the velocity of propagation of the longitudinal deformation wave.

The initial conditions determine the state of the rods before they collide; when $t = t_0 = 0$

$$u_{1t}(x, t_0) = V_0, \quad u_{1x}(x, t_0) = u_{2t}(x, t_0) = u_{2x}(x, t_0) = u_{3t}(x, t_0) = u_{3x}(x, t_0) = 0 \tag{1.2}$$

The boundary conditions determine that there is no force in the section $x = 0$ and that the velocity of the section $x = l$ is equal to zero during the interaction of segment *3* of the inhomogeneous rod with the rigid barrier:

$$u_{1x}(0, t) = u_{3t}(l, t) = 0 \tag{1.3}$$

and, also, determines the equality of the forces and velocities in the impacting sections $x = l_1$ of the homogeneous rod *1* and the initial segment *2* of the stepped rod when they directly interact

$$\begin{aligned} &EA_1 u_{1x}(l_1, t) = EA_2 u_{2x}(l_1, t), \quad \text{if} \quad u_{1x}(l_1, t) < 0 \\ &u_{1t}(l_1, t) = u_{2t}(l_1, t), \quad \text{if} \quad u_{1x}(l_1, t) < 0 \end{aligned} \tag{1.4}$$

or when there are no forces in the impacting sections of the rods if there is no interaction:

$$u_{1x}(l_1, t) = 0, \quad u_{2x}(l_1, t) = 0, \quad \text{if} \quad u_1(l_1, t) - u_2(l_1, t) \le 0 \tag{1.5}$$

Here, E is the modulus of elasticity of the first kind, A_1 is the cross-section area of the uniform rod *1* and A_2 is the cross-section area of the initial segment *2* of the stepped rod.

In the transitional cross-section $x = l_1 + l_2$ of the initial and final segments of the stepped rod, the boundary conditions also determine the equality of the forces and velocities

$$EA_2 u_{2x}(l_1 + l_2, t) = EA_3 u_{3x}(l_1 + l_2, t), \quad u_{2t}(l_1 + l_2, t) = u_{3t}(l_1 + l_2, t) \tag{1.6}$$

where A_3 is the cross-section area of the final segment *3* of the stepped rod.

2. Method of solution. Results

The differential Eq. (1.1) is solved by d'Alembert's method in the form[1]

$$u_n(x, t) = f_n(at - x) + \varphi_n(at + x)$$

$$u_{nx}(x, t) = -f_n'(at - x) + \varphi_n'(at + x), \quad u_{nt} = a[f_n'(at - x) + \varphi_n'(at + x)]$$

where $f_1(at - x)$, $f_2(at - x)$ and $f_3(at - x)$ are functions that describe the direct waves which propagate through segments *1, 2* and *3* respectively in the direction of the x axis, and $\varphi_1(at + x)$, $\varphi_1(at + x)$ and $\varphi_3(at + x)$ are functions describing the return waves which propagate through segments *1, 2* and *3* in the opposite direction. Derivatives of functions are denoted by a prime and the values of n are determined by the last formula of (1.1).

We will now change to relative quantities characterizing the direct and return waves, the deformation in a section and its velocity

$$\bar{f}'(at - x) = f'(at - x)(V_0/a)^{-1}, \quad \bar{\varphi}'(at + x) = \varphi'(at + x)(V_0/a)^{-1}$$

$$\bar{\varepsilon}(x, t) = -\bar{f}'(at - x) + \bar{\varphi}'(at + x), \quad \bar{v}(x, t) = v(x, t)V_0^{-1} = \bar{f}'(at - x) + \bar{\varphi}'(at + x)$$

Example 1. Consider a longitudinal collision between the uniform and stepped rods with lengths of the segments: $l_1 = 0.2l$ and $l_2 = l_3 = 0.4l$. The ratio of the cross-section areas of each preceding segment to the next one: $\lambda = A_1/A_2 = A_2/A_3 = 3$. In this case, the longitudinal stiffness of the cross-sections decreases in the direction of the rigid barrier. The field of wave states (Fig. 2) is constructed by the method of characteristics. Particular attention is paid to the existence of repeated collisions between the rods in the section $x = l_1$ and to the existence of repeated impacts of the stepped rod in the section $x = l$ with the barrier. The results of the modelling are shown in Table 1, where $\bar{t} = t(l/a)^{-1}$.

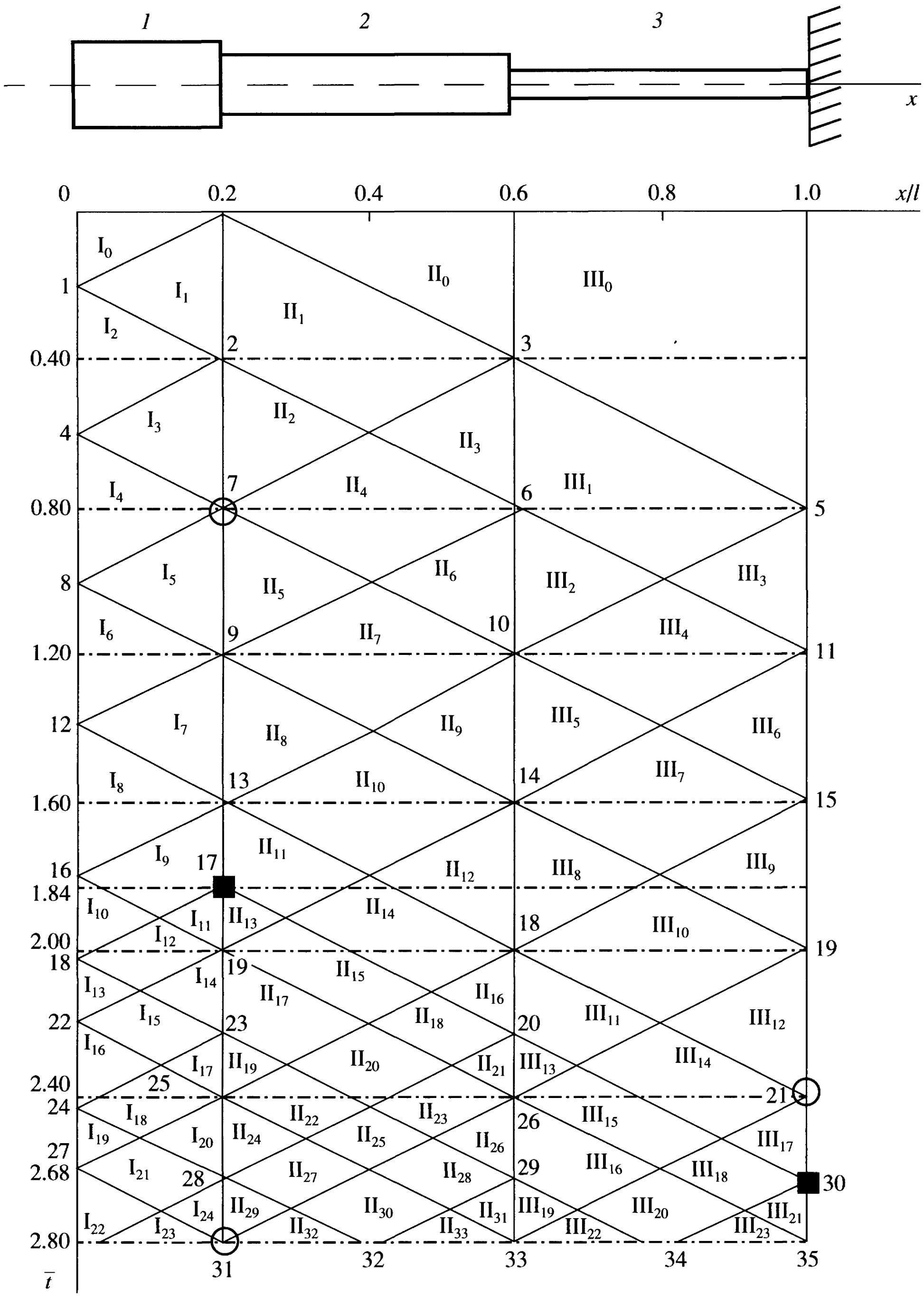

Fig. 2.

At the instant corresponding to the value of the relative time $\bar{t} = 0.8$, a direct wave $\bar{f}_1'(at - l_1) = 0.13$ (lines *4 - 7*, Fig. 2), formed in the section $x = 0$ when $\bar{t} = 0.6$ impinges on the section $x = l_1$ from the left. At the same instant, a new return wave $\bar{\varphi}_2'(at + l_1) = 0.38$, formed in the section $x = 0.6l$ when $\bar{t} = 0.4$ (lines *3 - 7*), approaches the section $x = l_1$ from the right. As a consequence of the unilateral constraint and in accordance with boundary conditions (1.3) - (1.6), the direct wave $\bar{f}_1'(at - l_1) = 0.13$ is reflected in the form of a return wave of the same magnitude $\bar{\varphi}_2'(at + l_1) = 0.13$ (lines *7 - 8*). In the zone of the fifth state of the first segment $\mathbf{I}_5$, the cross-sections of this segment will be spanned by these waves. In the corresponding sections of the first segment, the relative longitudinal deformation will be equal to zero and the velocity, including in the contact section $x = l_1$ (a uniform rod), is equal to $\bar{v}_2(l_1, t) = 0.26$. The duration of this state is determined for an arbitrary section of the first segment by the difference between the ordinates $\bar{t}$ which have the points of line *8 - 9* and the line *7 - 8* for this section.

Correspondingly, the wave returning from the right $\bar{\varphi}_2'(at + l_1) = 0.38$ which is incident on the section is reflected without any changes in the form of a direct wave $\bar{f}_2'(at - l_1) = 0.38$ (lines *7 - 10*). In the $\mathbf{II}_5$ domain, the cross-section of this segment will be spanned by these perturbations. In the corresponding sections of the second segment, the relative longitudinal deformation will be equal to zero and the velocity, including that in the contact section $x = l_1$, is equal to $\bar{v}_2(l_1, t) = 0.76$. The duration of this state for an arbitrary section of the second segment is determined by the difference between the ordinates t, which are the points of the line *6 - 9* and the line *7 - 10* for this segment.

Table 1

State domains	$\bar{f}'$	$\bar{\varphi}'$	$\bar{\varepsilon}$	$\bar{v}$	State domains	$\bar{f}'$	$\bar{\varphi}'$	$\bar{\varepsilon}$	$\bar{v}$
I_0	0.50	0.50	0.00	1.00	I_1	0.50	0.25	−0.25	0.75
I_2	0.25	0.25	0.00	0.50	I_3	0.25	0.13	−0.12	0.38
I_4	0.13	0.13	0.00	0.26	I_5	0.13	0.13	0.00	0.26
I_6	0.13	0.13	0.00	0.26	I_7	0.13	0.13	0.00	0.26
I_8	0.13	0.13	0.00	0.26	I_9	0.13	0.13	0.00	0.26
I_{10}	0.13	0.13	0.00	0.26	I_{11}	0.13	−0.13	−0.26	0.00
I_{12}	0.13	−0.13	−0.26	0.00	I_{13}	−0.13	−0.13	0.00	−0.26
I_{14}	0.13	−0.03	−0.16	0.10	I_{15}	−0.13	−0.03	0.10	−0.16
I_{16}	−0.03	−0.03	0.00	−0.06	I_{17}	−0.13	−0.16	−0.03	−0.29
I_{18}	−0.03	−0.16	−0.13	−0.19	I_{19}	−0.16	−0.16	0.00	−0.32
I_{20}	−0.03	−0.40	−0.37	−0.43	I_{21}	−0.16	−0.40	−0.24	−0.56
I_{22}	−0.40	−0.40	0.00	−0.80	I_{23}	−0.40	−0.46	−0.06	−0.86
I_{24}	−0.16	−0.46	−0.30	−0.62					
II_0	0.00	0.00	0.00	0.00	II_1	0.75	0.00	−0.75	0.75
II_2	0.38	0.00	−0.38	0.38	II_3	0.75	0.38	−0.37	1.13
II_4	0.38	0.38	0.00	0.76	II_5	0.38	0.38	0.00	0.76
II_6	0.38	0.19	−0.19	0.57	II_7	0.38	0.19	−0.19	0.57
II_8	0.19	0.19	0.00	0.38	II_9	0.38	−0.38	−0.76	0.00
II_{10}	0.19	−0.38	−0.57	−0.19	II_{11}	−0.38	−0.38	0.00	−0.76
II_{12}	0.19	−0.19	−0.38	0.00	II_{13}	0.39	−0.38	−0.77	0.01
II_{14}	−0.38	−0.19	0.19	−0.57	II_{15}	0.39	−0.19	−0.58	0.20
II_{16}	−0.38	−0.76	−0.38	−1.14	II_{17}	0.29	−0.19	−0.48	0.10
II_{18}	0.39	−0.76	−1.15	−0.37	II_{19}	−0.10	−0.19	−0.09	−0.29
II_{20}	0.29	−0.76	−1.05	−0.47	II_{21}	0.39	−0.37	−0.76	0.02
II_{22}	−0.10	−0.76	−0.66	−0.86	II_{23}	0.29	−0.37	−0.66	−0.08
II_{24}	0.34	−0.76	−1.10	−0.42	II_{25}	−0.10	−0.37	−0.27	−0.47
II_{26}	0.29	−0.14	−0.43	0.15	II_{27}	0.34	−0.37	−0.71	−0.03
II_{28}	−0.10	−0.14	−0.04	−0.24	II_{29}	−0.06	−0.37	−0.31	−0.43
II_{30}	0.34	−0.14	−0.48	0.20	II_{31}	−0.10	−0.34	−0.24	−0.44
II_{32}	−0.06	−0.14	−0.48	−0.20	II_{33}	0.34	−0.34	−0.68	0
III_0	0.00	0.00	0.00	0.00	III_1	1.13	0.00	−1.13	1.13
III_2	0.57	0.00	−0.57	0.57	III_3	1.13	−1.13	−2.26	0.00
III_4	0.57	−1.13	−1.70	−0.56	III_5	1.14	−1.13	−2.27	0.01
III_6	0.57	−0.57	−1.14	0.00	III_7	1.14	−0.57	−1.71	0.57
III_8	0.57	−0.57	−1.14	0.00	III_9	1.14	−1.14	−2.28	0.00
III_{10}	0.57	−1.14	−1.71	0.00	III_{11}	−0.01	−1.14	−1.13	−1.15
III_{12}	0.57	−0.57	−1.14	0.00	III_{13}	1.14	−1.14	−2.28	0.00
III_{14}	−0.01	−0.57	−0.56	−0.58	III_{15}	1.14	−0.57	−1.71	0.57
III_{16}	0.72	−0.57	−1.29	0.15	III_{17}	−0.01	−0.01	0.00	−0.02
III_{18}	1.14	−0.01	−1.15	1.13	III_{19}	0.14	−0.57	−0.71	−0.43
III_{20}	0.72	−0.01	−0.73	0.71	III_{21}	1.14	−1.14	−2.28	0.00
III_{22}	0.14	−0.01	−0.15	0.13	III_{23}	0.72	−1.14	−1.86	−0.42

It should be noted that $\bar{v}_1(l_1, t) < \bar{v}_2(l_1, t)$, and therefore the uniform rod detaches from the stepped rod in the contact section $x = l_1$. This detachment is shown by a circle in Fig. 2. As a result, the ends of the two rods in this section become free (Fig. 3, a).

The relative distance between the rods $\delta(t) = \delta(t)/l$ (Fig. 3, a) depends on the difference between the relative velocities of the contact sections $x = l_1$ and on the relative time $\bar{t} = t(l/a)^{-1}$

$$\bar{\delta}(t) = [\bar{v}_2(l_1, t) - \bar{v}_1(l_1, t)]\bar{t}$$

In the interval $0.8(l/a) \le t \le 1.2(l/a)$, we have $\bar{v}_2(l_1, t) - \bar{v}_1(l_1, t) = 0.50$. During the corresponding time $\bar{t} = 1.2 - 0.8 = 0.4$, the relative distance $\bar{\delta}_1 = 0.50 \times 0.4 = 0.20$. A diagram of the velocities of the sections of the uniform rod and the stepped rod when $t = 0.8l/a$ is shown in

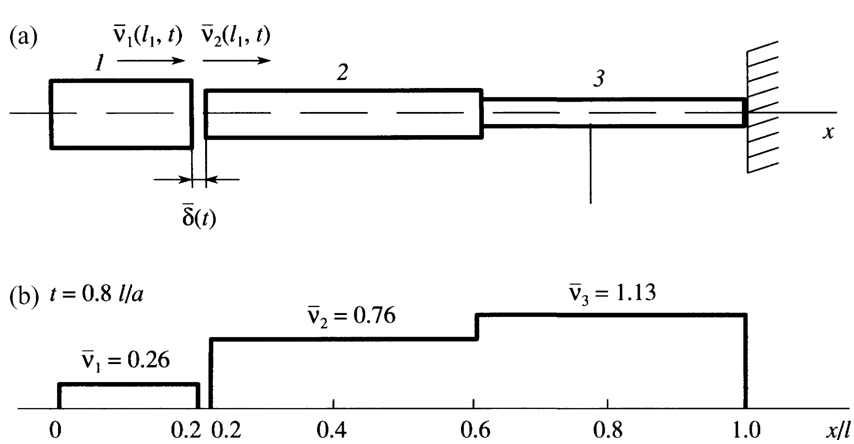

Fig. 3.

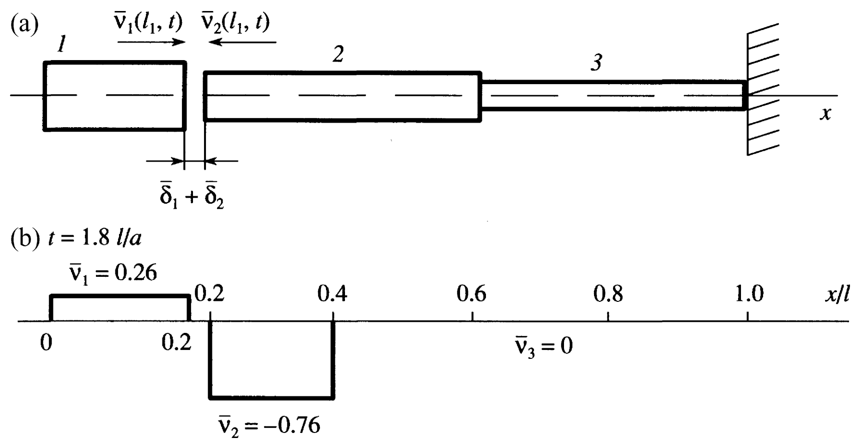

Fig. 4.

Fig. 3, *b*. In the first segment, the relative velocity of all the sections at this instant is $\bar{v}_1(x, t) = 0.26$, in the second segment it is $\bar{v}_2(x, t) = 0.76$ and, in the third segment, $\bar{v}_3(x, t) = 1.13$.

When $\bar{t} = 1.2$, in segment 2, a return wave $\bar{\varphi}_2'(at + l_1) = 0.19$, formed in the section $x = 0.8l$ (line *6 - 9*, Fig. 2), approaches the contact section $x = l_1$ from the right. This wave is reflected in the form of a direct wave $\bar{f}_1'(at - l_1) = 0.19$ from the section as from a free end. Then, in the domain of the eighth state of the second segment $\mathbf{II}_8$, the relative velocity of the contact section of the stepped rod is equal to $\bar{v}_2(l_1, t) = 0.38$. This wave state is maintained up to the instant $\bar{t} = 1.6$. At the same time, the velocity of the contact section of the uniform rod remains equal to $\bar{v}_1(l_1, t) = 0.26$. In this case, when $1.2 \le \bar{t} \le 1.6$, the relative distance between the rods increases by an amount $\bar{\delta}_2 = 0.048$.

When $\bar{t} = 1.6$, the free section of the stepped rod $x = l_1$ acquires a velocity $\bar{v}_2(l_1, t) = -0.76$ and begins to move to the left (Fig. 4*a* and *b*) since, at this instant, a return wave $\bar{\varphi}'(at + 0.6l) = -0.38$ impinges on it from the right (line *10 - 13*, Fig. 2) and is reflected in the form of a direct wave of the same magnitude (line *13 - 18*). The free section $x = l_1$ of the uniform rod keeps moving to the right with a velocity $\bar{v}_1(l_1, t) = 0.26$ (Fig. 4*a* and 4*b*). For this reason, the relative distance $\bar{\delta}(t)$ between the rods starts to contract and a second collision of the rods in the contact section is inevitable.

When $\bar{t} = 1.6$, the relative distance between the rods will be equal to $\bar{\delta}_1 + \bar{\delta}_2 = 0.248$. Consequently, for a second collision the contact sections of the rods must surmount this distance. This occurs over a time interval

$$\tau_1 = \frac{\bar{\delta}_1 + \bar{\delta}_2}{\bar{v}_1(l_1, t) + \bar{v}_2(l_1, t)} \approx 0.24\frac{l}{a}$$

from the instant $t = 1.6l/a$. Hence, a second collision of the rods in the contact section occurs at the instant $t = 1.6l/a + \tau_1 = 1.84l/a$. The second impact is shown by a dark square in Fig. 2. In accordance with the boundary conditions (1.3) - (1.6), the two rods are coupled in the contact section $x = l_1$ after the second collision. Hence, a new direct wave $\bar{f}_2'(at - l_1) = 0.39$ is formed from the right in the contact section (line *17 - 20*) and a new return wave $\bar{\varphi}_2'(at + l_1) = -0.13$ (line *17 - 18*) is formed from the left.

In the domain $\mathbf{II}_{13}$, the return wave coming from the right $\bar{\varphi}_2'(at + l_1) = -0.38$ (line *10 - 13*) and the new direct wave $\bar{f}_2'(at - l_1) = 0.39$ (line *17 - 20*) act on the cross-sections of the second segment. The relative longitudinal deformation in the corresponding sections of the second segment $\bar{\varepsilon}_2(x + t) = -0.77$ and the relative velocity of the sections in this segment $\bar{v}_2(x, t) = 0.01$. The duration of this state for an arbitrary section of the second segment is determined by the difference between the ordinates $\bar{t}$ which are the points of the lines *17 - 20* and *16–19* for this section.

In the domain I_{11}, direct wave from the left $\bar{f}_1'(at - l_1) = 0.13$ (line *12 - 13*) and a new return wave $\bar{\varphi}_1'(at + l_1) = -0.13$ (line *17 - 18*) act on the cross-sections. The relative longitudinal deformation in the corresponding sections of the second segment $\bar{\varepsilon}_1(x, t) = 0$ and the relative velocity of the sections in this segment $\bar{v}_1(x, t) = -0.26$. The duration of this state for an arbitrary section of the first segment is determined by the difference between the ordinates $\bar{t}$ which are the points of the lines *17 - 18* and *16 - 19* for this section.

When $\bar{t} = 2.8$, a direct wave $\bar{f}_1'(at - l_1) = -0.40$ (line *27 - 31*), which has been formed in the section $x = 0$ when $\bar{t} = 2.6$ impinges on the contact section $x = l_1$. When $\bar{t} = 2.8$, a return wave $\bar{\varphi}_2'(at + l_1) = -0.14$ (line *26 - 30*) impinges from the right on the contact section $x = l_1$. As a consequence of the unilateral constraint and in accordance with boundary conditions (1.3) - (1.6), the direct wave $\bar{f}_1'(at - l_1) = -0.40$ is reflected in the form of a return wave of the same magnitude $\bar{\varphi}_1'(at + l_1) = -0.40$. Correspondingly, the return wave $\bar{\varphi}_2'(at + l_1) = -0.14$, which is incident on the section from the right, is reflected unchanged in the form of a direct wave $\bar{f}_2'(at - l_1) = -0.14$. The velocity of the contact section in segment 1 will be equal to $\bar{v}_1(l_1, t) = -0.80$. The velocity of the section $x = l_1$ in segment 2 $\bar{v}_2(l_1, t) = -0.28$. Since the condition $\bar{v}_1(l_1, t) < \bar{v}_2(l, t)$ holds, detachment of the uniform and stepped rods occurs again in the contact section $x = l_1$. The detachment when $\bar{t} = 2.8$ is shown in Fig. 2 by a small circle. The parameters of the functions of the direct and return waves, as well as the relative longitudinal deformation $\bar{\varepsilon}(x, t)$ and the relative velocity $\bar{v}(x, t)$ for the corresponding sections of the domains of the states I_{23} and $\mathbf{II}_{32}$ including those for the contact section $x = l_1$, are shown in Table 1.

The relative distance between the rods is: $\bar{\delta}(t) = [\bar{v}_2(l_1, t) - \bar{v}_1(l_1, t)]\bar{t}$ (Fig. 5,*a*). When $\bar{t} = 2.9$, the cross-sections in the interval $0.1l < x < 0.2l$ have a relative velocity $\bar{v}_1(x, t) = -0.80$ and the cross-sections in the interval $0.2l < x < 0.3l$ acquire a relative velocity $\bar{v}_2(x, t) = -0.28$ (diagram of $\bar{v}$ in Fig. 5, *b*).

According to the results of the modelling, repeated collisions of the rods in the contact section are no longer noted and the relative distance $\bar{\delta}(t)$ will increase.

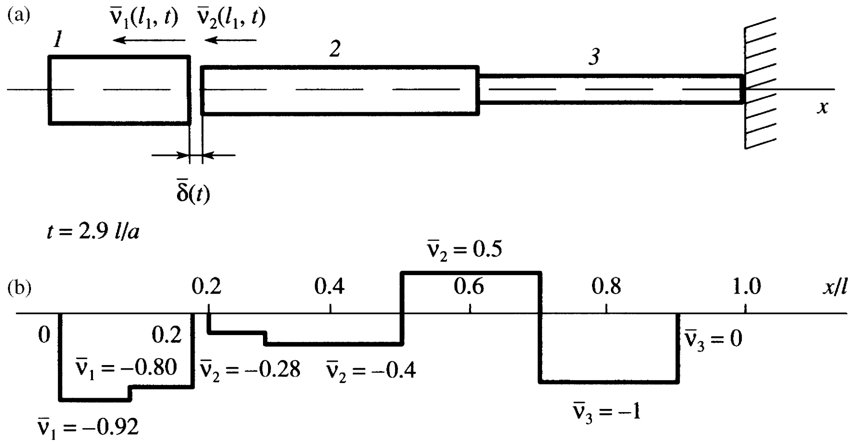

Fig. 5.

Table 2

l_1/l	l_2/l	l_3/l	$\lambda = 2$	$\lambda = 3$	$\lambda = 0.5$	$\lambda = 0.33$
0.2	0.2	0.6	1	2	0	0
0.2	0.4	0.4	2	2	0	0
0.2	0.6	0.2	2	3	0	0
0.4	0.2	0.4	3	1	0	0
0.4	0.4	0.2	3	2	2	0
0.6	0.2	0.2	0	1	0	0

The longitudinal collision was modelled for the different segment lengths l_1, l_2, l_3 shown below. The ratios λ of the cross-section areas of each preceding segment to the following segment are: $\lambda = 2$, $\lambda = 3$, $\lambda = 0.5$, $\lambda = 0.33$. It has already been pointed out above that particular attention was paid to the existence of repeated collisions and the elucidation of the laws governing their occurrence. The results of the modelling are presented in Table 2, where the overall number of repeated collisions between the rods in the section $x = l_1$ and between the stepped rod and the rigid barrier in the section $x = l$ during impact are shown.

It can be seen from Table 2 that, as the longitudinal stiffness decreases ($\lambda \geq 1$), there is an increase in the overall number of repeated collisions in the impact process and, when the longitudinal stiffness increases ($\lambda \leq 1$), there are practically no repeated collisions.

The need to take account of repeated collisions during the process of longitudinal impact is due to the following. At the instant of a repeated collision, new direct and return waves arise in the contact section which, on propagating from this section, form new wave states, which are characterized by new amounts of deformation and velocities of the sections of the segments. In certain cases after repeated collisions, the magnitude of the maximum longitudinal deformation ε in a number of segments of the rod system can be many times greater than the maximum longitudinal deformation prior to the break of contact, after which a repeated collision followed.[4] Taking account of repeated collisions can increase the overall duration of a longitudinal impact process T by a large factor.

Diagrams of the values of the duration of the impact process T and the maximum relative longitudinal deformation $\bar{\varepsilon}$ for rod systems without taking account of repeated collisions (the light tone in the diagram, T_1 and $\bar{\varepsilon}_I$) and when they are taken into account (the hatched tone in the diagram, T_{II} and $\bar{\varepsilon}_{II}$) have been constructed in Fig. 6 for the following combinations of the geometrical parameters

Figure	6, a	6, b	6, c	6, d	6, e	6, f
l_1/l	0.2	0.2	0.4	0.2	0.2	0.4
l_2/l	0.6	0.6	0.2	0.4	0.6	0.4
l_3/l	0.2	0.2	0.4	0.4	0.2	0.2
λ	2	3	3	3	2	0.5
T_{I}	1.2	1.2	3.0	0.5	0.85	0.75
T_{II}	6.0	6.0	0.5	2.6	1.41	1.5
$\bar{\varepsilon}_{\text{I}}$	1.08	1.08	0.8	1.1	1.1	0.57
$\bar{\varepsilon}_{I1}$	2.29	2.24	1.6	2.16	2.4	0.68

Example 2. We will now analyse the impact of a uniform rod of length $l_1 = 0.2l$ on a stepped rod interacting with a rigid barrier. The length of the first uniform segment of the stepped rod is $l_2 = 0.6l$ and the length of the second $l_3 = 0.2l$, $\lambda = 3$. It can be seen from the diagram that, when no consideration is given to the possibility of the occurrence of repeated collisions in sections with unilateral constraints, we have $T_I = 1.2l/a$, $\bar{\varepsilon}_I = 1.08$ (the Blank blocks, Fig. 6, *b*). If the impact process is tracked taking account of the repeated collisions which occur and the subsequent interruption of the contacts, then the duration of the impact T_{II} is $6.0l/a$ and the modulus of $\bar{\varepsilon}_{II}$ after a repeated impact, since new wave states arise, will attain a magnitude of 2.29 (the hatched blocks, Fig. 6, *b*). Hence, the duration of the loaded state of the rod system increases by a factor of five and the maximum longitudinal deformation becomes more than double. When there are repeated closures of the contacts during the longitudinal impact of rod systems, the quantities T and $\bar{\varepsilon}$ increase by one and a half to five times (see the diagrams in Fig. 6, *a, c, d, e, f*). It should be noted that repeated collision in sections with unilaterel constraints only occur in the case of a specific configuration of rod systems, that is, for a specific ratio of the lengths of the uniform segments and their cross-section areas.

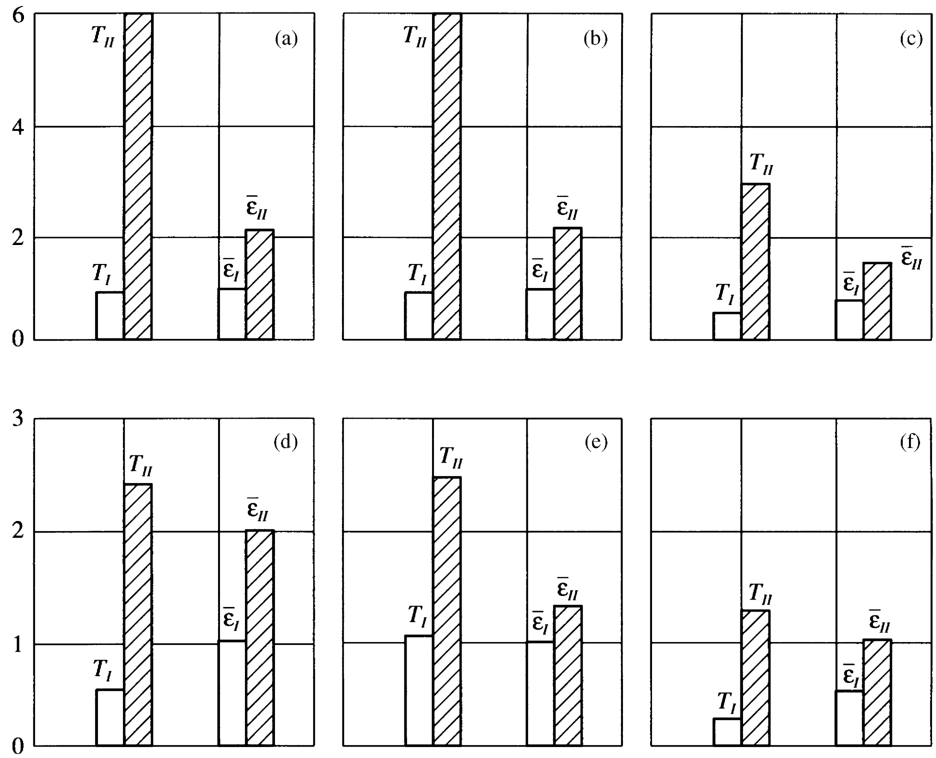

Fig. 6.

References

1. Aleksandrov YeV, Sokolinskii VB. *Applied Theory and the Calculation of Impact Systems*. Moscow: Nauka; 1969.
2. Alimov OD, Manzhosov VK, Yerem'yants VE. *Propagation of Strain Waves in Impact Systems*. Moscow: Nauka; 1985.
3. Bityurin AA, Manzhosov VK. Modelling of the longitudinal impact of uniform rods with unilateral constraints. *Vestn UlGTU* 2005;**3**:23–5.
4. Bityurin AA, Manzhosov VK. The change of deformation in segments of a rod system after repeated impact in the contact section. *Vestn UlGTU* 2007;**3**:23–8.
5. Veklich NA, Malyshev BM. The duration of the impact of an elastoplastic rod. *Izv Akad Nauk SSSR MTT* 1976;**2**:193–7.
6. Veklich NA, Malyshev BM. Longitudinal impact of a rigid body against a fixed rod. *Izv Akad Nauk SSSR MTT* 1976;**6**:140–6.
7. Malishev BM. The stability of rods under impact loading. *Izv Akad Nauk SSSR MTT* 1966;**4**:137–42.
8. Ionov VN, Ogibalov PM. *The strength of Spatial Structural Elements. Part 3. Dynamics and Stress Waves*. Moscow: Vysshaya Shkola; 1980.

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